



A quantitative method for the characterisation of karst aquifers based on spring hydrograph analysis

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Abstract

This paper presents a method for characterizing flow systems in karst aquifers by acquiring quantitative information about the geometric and hydraulic aquifer parameters from spring hydrograph analysis. Numerical sensitivity analyses identified two fundamentally different flow domains, depending on the overall configuration of aquifer parameters. These two domains have been quantitatively characterized by deducing analytical solutions for the global hydraulic response of simple two-dimensional model geometries.

During the baseflow recession of mature karst systems, the hydraulic parameters of karst conduits do not influence the drainage of the low-permeability matrix. In this case the drainage process is influenced by the size and hydraulic parameters of the low-permeability blocks alone. This flow condition has been defined as matrix-restrained flow regime (MRFR). During the baseflow recession of early karst systems and fissured systems, as well as the flood recession of mature systems, the recession process depends on the hydraulic parameters and the size of the low-permeability blocks, conduit conductivity and the total extent of the aquifer. This flow condition has been defined as conduit-influenced flow regime (CIFR).

Analytical formulae demonstrated the limitations of equivalent models. While equivalent discrete-continuum models of early karst systems may reflect their real hydraulic response, there is only one adequate parameter configuration for mature systems that yields appropriate recession coefficient. Consequently, equivalent discrete-continuum models are inadequate for simulating global response of mature karst systems. The recession coefficient of equivalent porous medium models corresponds to the transition between matrix-restrained and conduit-influenced flow. Consequently, equivalent porous medium models yield corrupted hydrographs both in mature and early systems, and this approach is basically inadequate for modelling global response of karst aquifers.

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1. Introduction

In order to be able to construct coherent distributive groundwater flow models of karst systems, the definition of realistic hydraulic and geometric

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parameters is essential (Király and Morel, 1976a; Király, 1998a, 2002). The lack of such data gives rise to significant problems in modelling karst hydrogeological systems.

A simple conceptual model of karst systems consists of a rectangular aquifer shape, a regular network of high-conductivity karst conduits embedded in the low-permeability fissured rock matrix, and a single karst spring that drains the conduit network. This model can be characterized by the hydraulic parameters of the low-permeability matrix and the conduit system, conduit spacing, and the spatial extent of the aquifer (Fig. 1).

Information on the hydraulic and geometric properties of karst systems can be obtained from classical geological and hydrogeological survey data, borehole tests, tracing experiments, speleological and geophysical observations and discharge measurements. However, these methods can provide only very limited information on the geometry and hydraulic properties of the conduit system.

In most cases some spring discharge time series data, coupled with information on the hydraulic properties of the low-permeability rock matrix are available. Although the global response of karst aquifers has been analyzed by several authors

(Maillet, 1905; Forkasiewicz and Paloc, 1967; Drogue, 1972; Mangin, 1975), the interpretation of this information has hitherto been mainly qualitative, and it has scarcely been used as a means of determining appropriate input data necessary for distributive modelling.

The aim of this paper is to quantitatively characterize the connection between the hydraulic and geometric properties of karst aquifers and their global response, in order to facilitate distributive groundwater flow modelling of karst systems. This has been achieved by deducing analytical solutions for the global hydraulic response of simple two-dimensional (2D) domains. The analytical formulae were then tested by numerical models. Resulting formulae express the connection between the hydraulic and geometric properties of a karst system and spring hydrograph recession coefficient, which is believed to be a characteristic parameter of the global response of a karst system.

2. Precedents

Every hydraulic process taking place in a karst aquifer manifests in the temporal variations in spring discharge. The plots of spring discharge versus time are referred to as spring hydrographs. Hydrographs consist of a succession of individual peaks, each of which represents the global response of the aquifer given to a precipitation event (Fig. 2). Hydrograph peaks consist of a rising and a falling limb. The rising limb comprises of a concave segment and a convex segment separated by an inflexion point. The falling limb comprises of a concave segment and a convex segment separated by an inflexion point.

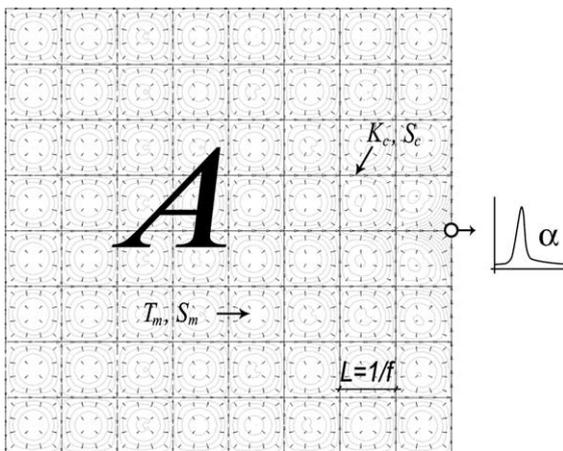


Fig. 1. A simple conceptual model suitable for the quantitative characterization of karst aquifers. T_m [$L^2 T^{-1}$] transmissivity of the low-permeability matrix, S_m [–] storativity of the low-permeability matrix, K_c [$L^3 T^{-1}$] 1D conduit conductivity, S_c [L] 1D conduit storativity, A [L^2] spatial extent of the aquifer, f [L^{-1}] frequency of karst conduits.

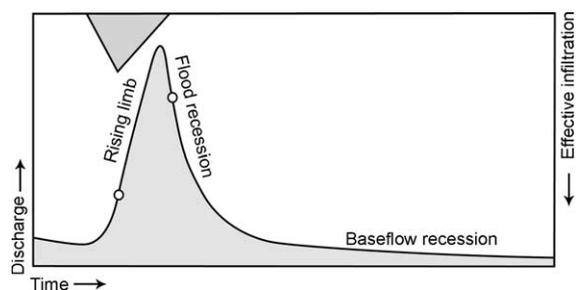


Fig. 2. Typical features of a spring hydrograph. White dots indicate inflexion points that correspond to the maximum infiltration state and to the end of the infiltration.

This inflexion point represents the maximum infiltration state (Király, 1998b). The falling limb comprises a steep and a slightly sloped segment. The former is called flood recession, while the latter is referred to as baseflow recession, which is the most stable section of any hydrograph. The flood recession limb is also divided into a convex segment and a concave segment by a second inflexion point, which represents the end of the infiltration event. Baseflow recession is the most representative feature of an aquifer's global response because it is the less influenced by the temporal and spatial variations of infiltration.

The first mathematical characterization of the baseflow recession was provided by Maillet (1905). This interpretation is based on the drainage of a simple reservoir, and presumes that the spring discharge is a function of the volume of water held in storage. This behavior is described by an exponential equation as follows

$$Q_{(t)} = Q_0 e^{-\alpha t} \quad (1)$$

where Q_t is the discharge [$L^3 T^{-1}$] at time t , and Q_0 is the initial discharge [$L^3 T^{-1}$], α is the recession coefficient [T^{-1}] usually expressed in days. On a semilogarithmic graph, this function is represented as a straight line with the slope $-\alpha$. This equation is usually adequate for describing baseflow recession of karst systems, and is believed to reflect the drainage of the saturated low-permeability fissured matrix.

Berkaloff (1967) provided a solution for diffusive flux from a one-dimensional (1D) conductive block having fix-head boundary condition at one edge, and steady-state initial conditions over the block. This may be expressed as follows

$$Q = \frac{2TR_0}{SL} \sum_{n=1}^{\infty} \exp\left(-\left(n - \frac{1}{2}\right)^2 \pi^2 \frac{Tt}{SL^2}\right) \quad (2)$$

where T is hydraulic transmissivity [$L^2 T^{-1}$], S is storativity [$-$], R_0 is constant recharge expressed as [L], and L is the length of the 1D block [L]. Neglecting the higher order terms of the series, the discharge can be approximated as:

$$Q_{(t)} = \frac{2TR_0}{SL} \exp\left(-\frac{\pi^2 Tt}{4SL^2}\right) \quad (3)$$

The 1D block recession coefficient is thus:

$$\alpha_b = \frac{\pi^2 T}{4SL^2} \quad (4)$$

Bagarić (1978) provided an analytical solution for diffusive flux from a 1D conductive block that has fix-head boundary condition at one edge. In contrast to the model of Berkaloff (1967), no infiltration is applied in this concept. According to Bagarić, the discharge of the block may be expressed as follows:

$$Q_{(t)} = Q_0 \exp\left(-\frac{2T}{SL^2} t\right) \quad (5)$$

Based on Eq. (5), the recession coefficient of a 1D block may be equated to transmissivity and storage as follows:

$$\alpha_b = \frac{2T}{SL^2} \quad (6)$$

This solution is quite similar to that of Berkaloff (1967) expressed by Eq. (4).

The above analytical formulae consider the recession process as exclusively dependent on the hydraulic parameters of the low-permeability matrix, and neglect the influence of the conduit network on the drainage process. Although mathematical formulation has not yet been provided which describes the influence of the conduit network on the recession coefficient, some sensitivity analyses have been performed to describe this process qualitatively.

The first distributive modelling code capable of combining conduit flow and diffuse flow was created by Király and Morel (1976a); Király (1985). These authors performed some sensitivity analyses (Király and Morel, 1976b) and concluded that the increase of the conduit network density results in higher baseflow recession coefficients. Additional sensitivity analyses were performed by Eisenlohr (1996); Eisenlohr et al. (1997), who constructed a series of 2D synthetic models having different karst network densities, and by Cornaton (1999) who investigated three-dimensional (3D) synthetic models. These simulations confirmed the results of Király and Morel (1976b). According to further simulations of Eisenlohr (1996), an increase of the conduit network conductivity resulted in a corresponding rise in baseflow recession coefficient. Moreover, the simultaneous increase of the storage coefficients of both the low-permeability

matrix and the karst conduit network resulted in the decrease of the recession coefficient. Cornaton (1999) expanded this analysis by investigating the separate effects of the variation of matrix or conduit storage within 3D synthetic models, and similarly concluded that an increase in either the matrix storage coefficient or conduit storage coefficient resulted in a lower recession coefficient.

Eisenlohr (1996) investigated the effects of the shape and duration of the recharge function on the hydrograph. This study demonstrated that until the duration of the recharge functions and the total infiltrations are identical, the baseflow recession coefficients are similar. However, the fast recession limb showed a strong variation. If the recharge function implies delayed infiltration, the baseflow recession coefficient decreases.

Eisenlohr (1996) also constructed synthetic models having the same conduit network density, but different orientations of the network. These simulations showed that the baseflow recession coefficient increases as the orientation of conduit network corresponds more closely to the model domain's longest orientation.

The above discussed studies demonstrate that the baseflow recession coefficient depends not only on the hydraulic properties of the low-permeability matrix, but also on the hydraulic and geometric parameters of the conduit system which are neglected by the models of Berkloff (1967); Bagarić (1978). Although the above-mentioned sensitivity analyses demonstrated the variations of the recession coefficient according to the varying aquifer properties, they comprised only a small number of simulations. Consequently, these simulations could provide only a qualitative indication of the influence of certain aquifer parameters on spring hydrographs.

3. An analytical solution for diffusive flux from a two-dimensional homogeneous square block

The analytical formulae described in Section 2 were based on 1D models. In order to provide a more realistic mathematical characterization of diffusive flux from a conductive block (Fig. 3), a 2D analytical solution has been derived from the heat flow equation solution of Carslaw and Jaeger (1959, pp. 173), using

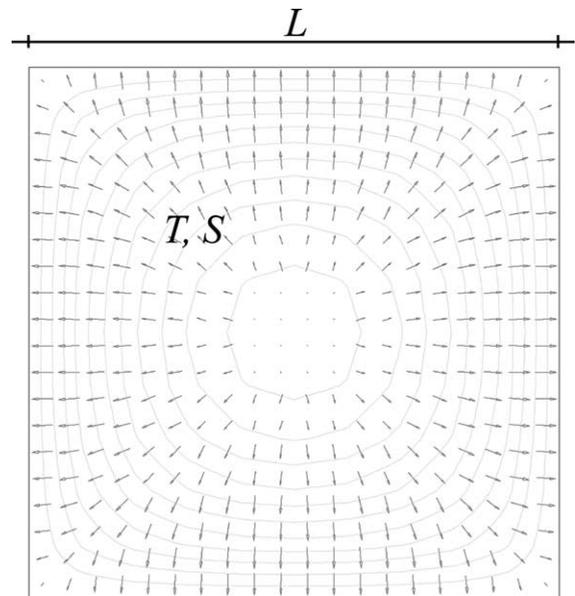


Fig. 3. Equipotential lines and flux vectors over a two-dimensional homogeneous block having encircling uniform head boundary. L [L] size of the domain, T[L²T⁻¹] hydraulic transmissivity, S[-] storativity.

the following formula

$$Q_{(t)} = \frac{128}{\pi^2} T \sum_{n=0}^{\infty} \exp\left(- (2n + 1)^2 \pi^2 \frac{Tt}{SL^2}\right) \times \sum_{n=0}^{\infty} \frac{\exp\left(- (2n + 1)^2 \pi^2 \frac{Tt}{SL^2}\right)}{(2n + 1)^2} \quad (7)$$

where uniform hydraulic heads are assumed as boundary conditions along the sides of the square block, and initial conditions comprise uniform hydraulic heads over the block surface. Assuming that

$$a = \frac{\pi^2 Tt}{SL^2} \quad (8)$$

it follows from Eq. (7) that

$$Q_{(t)} = \frac{128}{\pi^2} T (e^{-a} + e^{-9a} + e^{-25a} + \dots) \times \left(e^{-a} + \frac{e^{-9a}}{9} + \frac{e^{-25a}}{25} + \dots \right) \quad (9)$$

Neglecting the higher order terms of the series (this has been verified by a series of numerical models

discussed in the following sections), the discharge can be approximated as

$$Q_{(t)} = \frac{128}{\pi^2} T \left(\exp \left(-2\pi^2 \frac{Tt}{SL^2} \right) \right) \quad (10)$$

Comparison of this solution with the classical formula of Maillet (1905) (Eq. (1)) shows that the recession coefficient of a 2D homogeneous block may be expressed as

$$\alpha_b = \frac{2\pi^2 T}{SL^2} \quad (11)$$

4. Effect of the alteration of conduit parameters on recession coefficient

The diffusive flux (Eq. (10)) from a homogeneous square block has been mathematically characterized by use of the recession coefficient defined by Eq. (11). The recession of an entire karst system can be determined using this approach if the conductive capacity of the water drainage system is assumed to be sufficiently high and the storage in the conduit network is neglected. Based on this approach, the discharging water from low-permeability blocks is assumed to reach the outlet of a system instantaneously.

In order to check the domain of validity for this simplification and the influence of changing conduit characteristics on the recession coefficient, numerical sensitivity analyses have been performed, using the combined discrete-continuum method (Király, 1979, 1985; Király and Morel, 1976a). This approach uses the finite element discretization scheme, which allows the combination of one-, two-, and three-dimensional elements. Consequently, high conductivity karst channels can be simulated by 1D finite elements, which are embedded in the low permeability matrix represented by two- or three-dimensional elements. Numerical simulations were performed making use of the computer codes FEN1 and FEN2 (Király, 1985).

With an initial set of selected simulations, the sensitivity of the recession coefficient to the conduit conductivity was investigated. A second series of simulations was performed in order to investigate the sensitivity of the recession coefficient to conduit frequency. Conduit storage coefficients S_c [L] were

calculated from relevant conduit apertures assuming water compression alone as follows

$$S_c = \frac{\rho g}{E_w} ab \quad (12)$$

where $E_w = 1/\beta_w$ is the bulk modulus of water compression [$\text{ML}^{-1}\text{T}^{-2}$] and b is fracture width, here assumed to be 1 m. The conduit aperture a [m] can be expressed from conduit conductivity K_c [L^3T^{-1}] by the ‘cubic law’ (Witherspoon et al., 1980) as follows:

$$K_c = \frac{a^3 b}{12} \frac{\rho g}{\mu} \quad (13)$$

In fact, in 2D models the 1D conduits behave similarly to 2D trenches in a 3D medium. Consequently, the above formula usually applied for fractures was used for estimating the conduit storage coefficients.

As demonstrated in Fig. 4, an increase in conduit conductivity resulted in a rise in recession coefficient, until reaching the value of the analytical solution for a single homogeneous block. By exceeding a threshold value, the increase of the conduit conductivity had no further influence on the baseflow recession coefficient, the recession process is controlled by the hydraulic parameters of the low-permeability blocks alone, and Eq. (11) provides an adequate characterization of the systems global response.

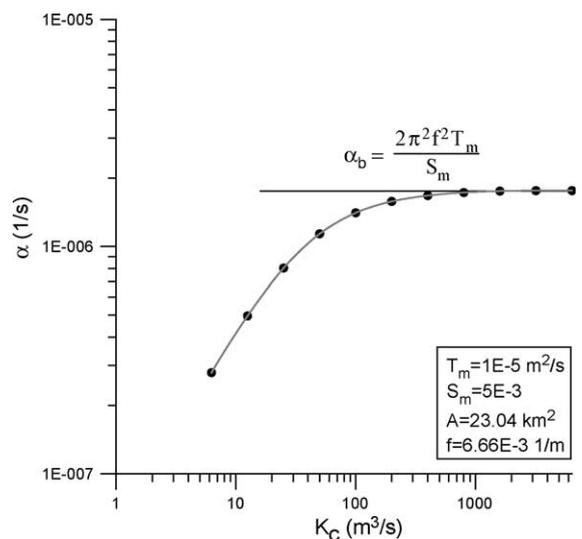


Fig. 4. Dependence of the recession coefficient on conduit conductivity. Simulation results are represented by dots.

This means, that for sufficiently high conduit conductivities, the further increase of this parameter does not influence the recession process, since the discharging capacity of the low-permeability blocks remains lower than the conductive capacity of the conduits. A threshold value of conduit conductivity must exist for every model configuration.

By exceeding this value, the hydraulic gradient in the conduits is negligible during the recession process, and conduit flow has no influence on the hydraulic gradient over the low-permeability blocks. The conduits act as fix-head boundary conditions as assumed by the above analytical model. This flow condition has been defined as *matrix-restrained flow regime* (MRFR), and the set of parameter configurations corresponding to such baseflow conditions is thus referred to as *matrix-restrained baseflow domain*.

If the conduit hydraulic conductivity is lower than the threshold value, the recession coefficient is strongly dependent on the conduit conductivity, and the analytical Eq. (11) is no longer valid. This flow condition has been defined as *conduit-influenced flow regime* (CIFR), and the set of parameter configurations corresponding to such baseflow conditions is referred to as *conduit-influenced baseflow domain*. This case is further investigated in Section 5.

The influence of changing conduit frequencies on the recession coefficient follows similar principles as

the alteration of conduit conductivity. However, the alteration of conduit frequency involves not only the change of the number of conductive features, but also the alteration of the low-permeability block size. Thus, although low frequency domains can be characterized by Eq. (11), any change of conduit frequency influences the value of the recession coefficient (Fig. 5). A threshold value of conduit frequency exists for every hydraulic parameter configuration, and above this threshold, the change of conduit frequency entails the change of the recession coefficient according to an unknown function to be developed in the following section.

5. Characterization of the recession of heterogeneous domains

Characterizing the link between heterogeneous domain recession coefficient and aquifer hydraulic and geometric parameters is the principal goal of this section. According to the previous sections, heterogeneous aquifers having sufficiently high contrast between the conductive capacity of the conduits and the conductive capacity of low permeability blocks (matrix-restrained baseflow domain) can be characterized by Eq. (11). The principles of the recession process in aquifers, where the system heterogeneity is insufficiently high (conduit-influenced baseflow domain) were investigated by assuming that an aquifer's global response can be approximated by the application of the equivalent porous medium concept, with further restrictions. The domains of validity of equivalent formulae were then tested and the formulae were corrected based on a series of sensitivity analyses.

5.1. Equivalent porous medium approach for evaluating recession coefficient in the conduit-influenced baseflow domain

By arranging the hydraulic parameters of a porous medium in a manner that an elementary volume of the aquifer transmits the same specific discharge and releases the same amount of water from storage as that of a heterogeneous domain, an equivalent porous medium can be made (Fig. 6).

Although the characterization of diffusive flux from a homogeneous domain having point-like head

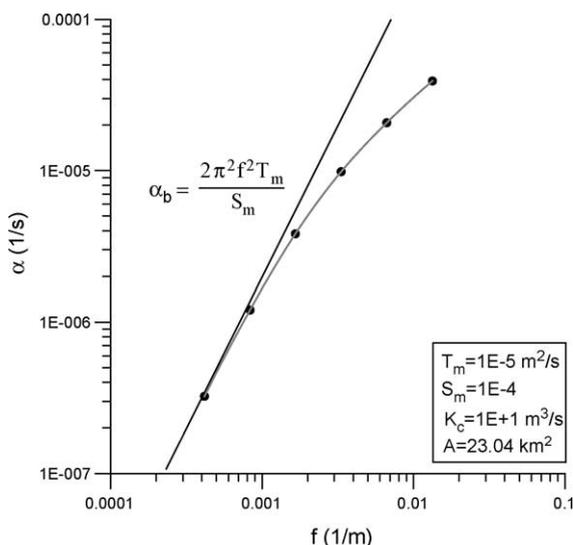


Fig. 5. Dependence of the recession coefficient on conduit frequency. Simulation results are represented by dots.

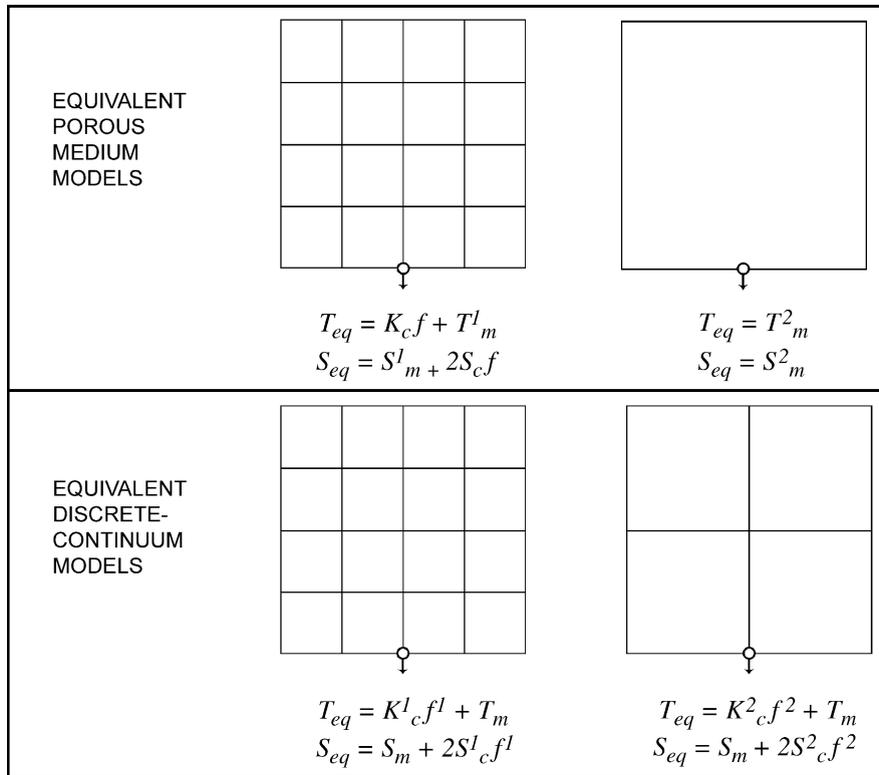


Fig. 6. Hydraulic parameters of equivalent porous medium and equivalent discrete-continuum models.

boundary condition is far simpler than that of a strongly heterogeneous domain, the lack of analytical formulae describing this former phenomenon necessitated the development of an empirical formula.

5.1.1. Diffusive flux from a homogeneous square domain

The empirical formula describing diffusive flux from a homogeneous domain having point-like head boundary condition was developed by constructing a variety of synthetic homogeneous models, altering hydraulic parameters, and subsequently deducing the equation that governs flux by fitting each parameter.

According to the simulation results (Kovács, 2003), the formulation of diffusive flux from a homogeneous domain with a point-like head boundary is similar to the formulation of diffusive flux from a homogeneous block with encircling head boundary, with a different geometric factor (γ). The formulation of recession coefficient related to diffusive flux from

a conductive block takes the following general form:

$$\alpha = \frac{\gamma T}{SA} \quad (14)$$

where γ is the geometric factor [–] dependent on domain shape and boundary conditions, and A is the domain area [L^2]. For 2D square flow domains with point-like head boundary condition, the geometric factor is $\gamma = 4/9$.

5.1.2. Diffusive flux from a heterogeneous domain

Supposing that the recession of a heterogeneous domain can be approached by the drainage of an equivalent porous domain of the same dimensions and same boundary conditions, the equivalent transmissivity parallel to karst conduits may be expressed as follows

$$T_{eq} = K_c f + T_m \quad (15)$$

where T_{eq} is equivalent transmissivity [$L^2 T^{-1}$], K_c is 1D conduit conductivity [$L^3 T^{-1}$], f is conduit

frequency (number of conduits per unit length) [L⁻¹], and T_m is the transmissivity of the matrix medium [L²T⁻¹].

Similarly, as there are two intersecting conduits for each block, the equivalent storage (S_{eq}) can be expressed in the following form

$$S_{eq} = S_m + 2S_c f \tag{16}$$

where S_m is the matrix storativity [-] and S_c is the 1D conduit storage coefficient [L].

Substituting the equivalent parameters into the formulation of the recession coefficient (Eq. (14)), the equivalent recession coefficient becomes

$$\alpha_h = \frac{\gamma (K_c f + T_m)}{A (S_m + 2S_c f)} \tag{17}$$

where the transmissivity of the low permeability matrix is usually several orders of magnitude lower than the equivalent conductivity of the conduit system. Similarly, in the case of phreatic karst systems, the equivalent storage of the conduits is much smaller than the storativity of the matrix. Consequently, the formula can be further simplified by neglecting these terms:

$$\alpha_h = \frac{\gamma (K_c f + T_m)}{A (S_m + 2S_c f)} \approx \gamma \frac{(K_c f + T_m)}{S_m A} \approx \gamma \frac{K_c f}{S_m A} \tag{18}$$

Because of the introduction of heterogeneity into the model, the geometric parameter γ is expected to differ from the empirically obtained value for porous equivalent domains. Furthermore, the formulation of the recession coefficient based on the porous equivalent medium concept should be tested on a large variety of discrete-continuum models. Consequently, a large number of numerical model simulations were performed. The results of these analyses are presented in Section 5.2.

5.2. Evaluation of the geometric parameter in the conduit-influenced baseflow domain

In order to test the validity of Eq. (18), several different numerical tests were performed. The effect of the systematic modification of each hydraulic and geometric parameter was investigated. These sensitivity analyses and the investigated parameter

configurations are explained in details in Kovács (2003).

Simulation results confirm the existence of two significantly different flow domains, previously explained in Section 4, for the alteration of any parameters. While the matrix-restrained baseflow domain can be described by the analytical Eq. (11), the conduit-influenced baseflow domain is approachable by the general formula provided in Section 5.1 (Eq. (18)). In order to test the validity of this formula, and to obtain the value of the geometric parameter γ , the simulation results falling into these two distinct baseflow domains were separated, and only the conduit-influenced baseflow domain was considered during the evaluation process. Among 82 flow simulations, 52 simulations were carried out in the conduit-influenced domain.

Rearranging the Eq. (18) in a dimensionless manner by normalizing by T_m yields

$$\begin{aligned} \alpha_h A \frac{(S_m + 2S_c f)}{T_m} &\approx \alpha_h A \frac{S_m}{T_m} \\ &= \gamma \left(\frac{K_c}{T_m} f + 1 \right) \approx \gamma \frac{K_c f}{T_m} \end{aligned} \tag{19}$$

where the value of γ is represented by the slope of the line fitted to the simulation results (Fig. 7).

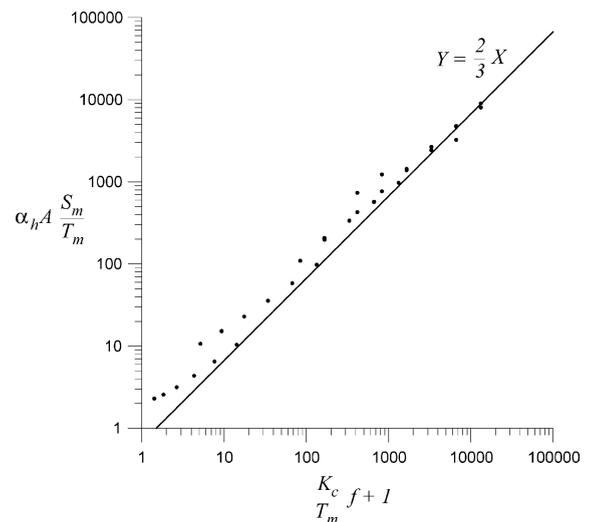


Fig. 7. Evaluation of the geometric parameter (γ) of conduit-influenced regular karst systems by the means of curve fitting to numerical model results.

Curve fitting yields the value of $\gamma=2/3$. Although slight deviations in simulation results from the fitted curve occur, the final formula for the recession coefficient of the conduit-influenced baseflow domain can be expressed as follows:

$$\alpha_h = \frac{2}{3} \frac{(K_{cf} + T_m)}{A(S_m + 2S_{cf})} \approx \frac{2}{3} \frac{(K_{cf} + T_m)}{S_m A} \approx \frac{2}{3} \frac{K_{cf}}{S_m A} \quad (20)$$

Eq. (20) provides a quite good approximation of the recession coefficient (Fig. 7). The slight deviation among some of the simulation results from the fitted line is very probably due to differences in model discretization.

The plot of recession coefficients calculated using Eq. (20) against the numerically obtained values is shown in Fig. 8. Eq. (20) underestimates the value of the recession coefficient for several parameter configurations. Its average error is about 25%, although the maximum error never exceeds 68% even for the most unrealistic parameter configurations. As the difference between the two extremes of simulated recession coefficients is more than 5 orders of magnitude and the formula error is in the same range as the natural variation of the recession coefficient, Eq. (20) gives a reasonable estimate of aquifer parameters.

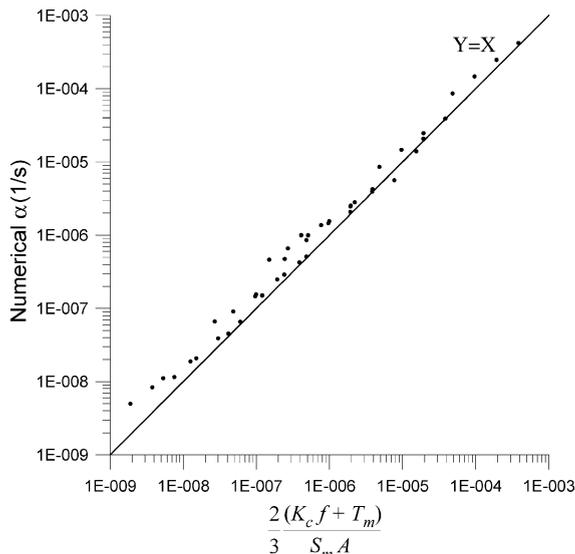


Fig. 8. Plot of analytically calculated recession coefficients against the numerically obtained values.

6. General mathematical characterization of the recession process

Previous sections have demonstrated that the alteration of aquifer hydraulic and geometric properties results in changing recession coefficients. The dependence of recession coefficient on aquifer properties follows two fundamentally different principles:

1. The MRFR flow regime is controlled by the hydraulic parameters of the low-permeability medium. This case can be mathematically characterized by the drainage of a homogeneous block (Eq. (11)).
2. The CIFR flow regime is mainly controlled by the conductive capacity of the conduit system. This case can be mathematically characterized by Eq. (20).

The two principal baseflow domains are linked by a transition zone in which recession coefficient follows neither of the corresponding functions, but a mathematically unspecified intermediate function. The existence of these two distinct flow domains is the manifestation of the ‘duality of groundwater flow field’ defined by Király (1994); Király (2002)) as the direct consequence of the heterogeneity of the hydraulic conductivity field. The heterogeneity of a karst system changes not only with the alteration of the contrasts in hydraulic conductivity, but also with the alteration of the frequency of high-conductivity features. Consequently, a transition between the two principal baseflow domains exists for the alteration of both of these parameters.

The threshold separating the two principal flow domains can be mathematically expressed by equating the matrix-restrained (Eq. (11)) and the conduit-influenced (Eq. (20)) recession coefficients. This can be expressed as follows

$$\alpha_b = \alpha_h \quad (21)$$

thus

$$\begin{aligned} \frac{2\pi^2 T_m f^2}{S_m} &= \frac{2}{3} \frac{(K_{cf} + T_m)}{A(S_m + 2S_{cf})} \\ &\approx \frac{2}{3} \frac{(K_{cf} + T_m)}{S_m A} \approx \frac{2}{3} \frac{K_{cf}}{S_m A} \end{aligned} \quad (22)$$

where the threshold value of conduit conductivity (K_c^*) neglecting the storage in karst conduits can be expressed as follows:

$$K_c^* = 3\pi^2 T_m A f - \frac{T_m}{f} \approx 3\pi^2 T_m A f \quad (23)$$

For smaller values of K_c the flux is mainly restrained by the conductive capacity of the high-permeability conduit network, while for higher values of K_c , flux is restrained by the diffusivity of the low-permeability matrix. The graphical representation of recession coefficients dependence on conduit conductivity is demonstrated in Fig. 9.

Similarly to the conductivity domain, a threshold value dividing the two principal flow domains can be defined in the frequency domain. This can be expressed as follows:

$$f^* = \frac{K_c + \sqrt{K_c^2 + 12\pi^2 A T_m^2}}{6\pi^2 T_m A} \approx \frac{K_c}{3\pi^2 T_m A} \quad (24)$$

where f^* is the threshold frequency dividing matrix-restrained and conduit-influenced baseflow domains. For smaller values of f , the recession is restrained by the diffusivity of the low-permeability matrix, while for higher values of f , recession is mainly restrained by the conductive capacity of the high-permeability

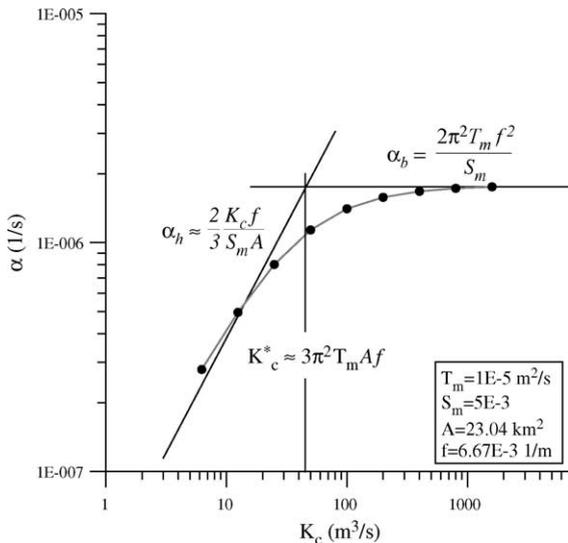


Fig. 9. Graphical representation of the dependence of the recession coefficient on conduit conductivity. Simulated recession coefficients (dots) can be approached by functions α_b and α_h . K_c^* represents the threshold value dividing these two flow domains.

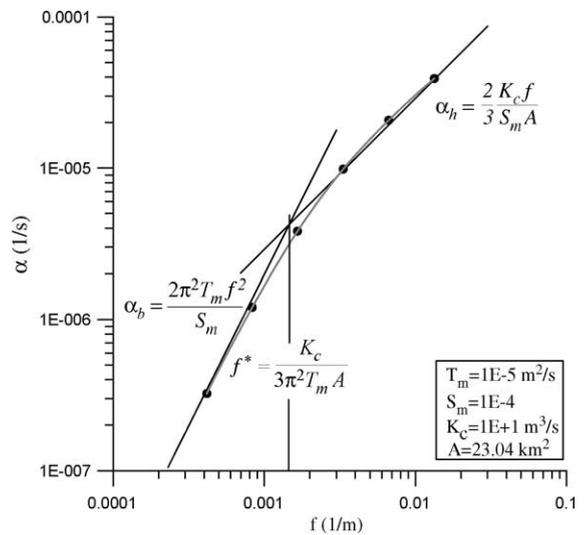


Fig. 10. Graphical representation of the dependence of the recession coefficient on conduit frequency. Simulated recession coefficients (dots), can be approached by functions α_b and α_h . f^* represents the threshold value dividing these two flow domains.

conduit network. The graphical representation of recession coefficient's dependence on conduit frequency is demonstrated in Fig. 10.

The evaluation of threshold parameters facilitates the qualitative classification of karst systems: while MRFR flow regime is a characteristic of *mature karst systems* under baseflow conditions, *early karst systems* and *fissured systems* exhibit CIFR baseflow.

The value of the recession coefficient corresponding to the threshold point and its physical meaning is discussed in Section 7.

7. Validity of equivalent models

The validity and applicability of equivalent models is a crucial question in modelling fissured and karst systems, not only because the estimation of the conduit hydraulic and geometric parameters is difficult, but also because the appropriate modelling code may not be available. Equivalent porous medium models are often used without adequate test of their applicability, and the application of equivalent discrete-continuum models may be a consequence of the false estimation of conduit frequency. The fundamental difference between equivalent porous

medium and equivalent discrete-continuum models is demonstrated in Fig. 6.

As it was explained in the previous sections, the principles governing flux from heterogeneous systems are far more complicated than those of which govern flux from homogeneous systems. While the recession of homogeneous systems always follows the same principles, the evaluation of the global response of heterogeneous systems requires the discussion of flow parameters and the selection of the appropriate equation. Furthermore, the dominant flow regime of heterogeneous systems may change throughout the recession process.

In order to demonstrate the variations of the recession coefficient for domains having the same equivalent hydraulic parameters but different conduit geometries and conduit conductivities, a series of numerical tests was performed on equivalent discrete-continuum models. The results of these simulations are shown in Fig. 11.

These numerical tests demonstrated that in the matrix-restrained baseflow domain the equivalent discrete-continuum models yield systematically

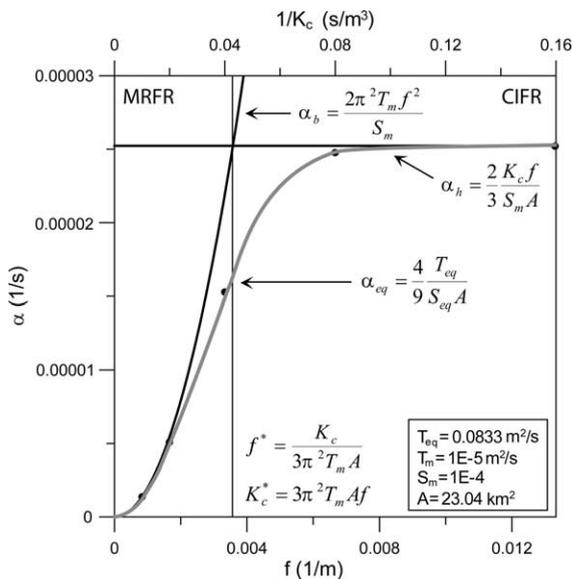


Fig. 11. Variations of the recession coefficient for equivalent discrete-continuum domains. Each model (results represented by black dots) has the same equivalent hydraulic parameters, but different conduit conductivities and frequencies. The recession coefficient of the porous equivalent medium model is identical to the recession coefficient of the threshold parameter discrete-continuum model, and designates an inflection point of the curve.

different recession coefficients, as they follow Eq. (11), which is dependent exclusively on block size and matrix hydraulic properties. In this baseflow domain none of the equivalent models has validity. In the conduit-influenced baseflow domain, all of the equivalent discrete-continuum models yield the same recession coefficient. Between these fundamentally different baseflow domains a transition zone exists. In the centre of this zone, the threshold parameters designate an inflection point, which coincides with the recession coefficient of the equivalent porous medium model. This means that heterogeneous systems having hydraulic and geometric parameters corresponding to the threshold values behave like equivalent porous medium. This is the only point of validity of the equivalent porous medium approach when simulating the recession of karst or fissured systems.

8. Graphical method for the estimation of aquifer parameters

The principles governing the recession process have been mathematically defined in previous sections. In order to facilitate the estimation of desired parameters, a graphical representation of parameter dependences is provided in Fig. 12.

Parameter dependences represented in this figure involve the negligence of storage related to the drainage of vertical shafts and variably saturated conduits. Furthermore, the equations represented in this figure were derived from 2D models, and their correct application requires the transformation of measured field parameters into 2D model parameters. For example, the conduit conductivity of multi-level karst systems must be multiplied by the number of active conduit horizons. Similarly, 2D transmissive and capacitive parameters of the low-permeability matrix must be specified respecting the average thickness of the saturated zone.

9. Conclusions

Spring hydrograph recession coefficient is a characteristic global parameter of an aquifer and supplies important information concerning the system's hydraulic and geometric parameters. As these

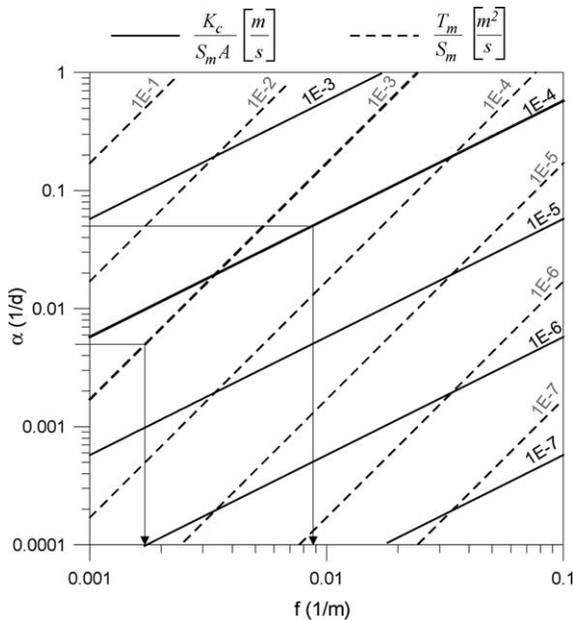


Fig. 12. Graphical representation of the relationship between aquifer hydraulic and geometric parameters and baseflow recession coefficient. Subsequently to the identification of the two intersecting lines corresponding to field measurements, the lowermost line must be selected for the evaluation of the desired parameter. For example the parameter set of $K_c=100$, $T_m=1 \times 10^{-5}$, $S_m=1 \times 10^{-2}$, $A=1 \times 10^8$, and the measured value of $\alpha=0.005$ 1/day yields $f=1.7 \times 10^{-3}$ 1/m, while $\alpha=0.05$ day yields $f=8.7 \times 10^{-3}$ 1/m.

parameters are rarely measurable, discharge time series represent a very important means of determining of information about the overall structure of a karst system. The drainage of a porous domain can be described by a general exponential model, with the following recession coefficient

$$\alpha = \frac{\gamma T}{SA}$$

where γ is a geometric factor that depends on boundary conditions and aquifer shape.

The drainage process of karst aquifers follows similar rules; however, there is a significant difference between porous and karst systems. While the drainage of porous systems is always dependent on the extent of the entire aquifer, the drainage of karst systems depends either on the extent of the entire aquifer or on the extent of individual low-permeability fissured blocks surrounded by karst conduits. The respective law that characterizes the drainage of a karst system

depends on the overall geometric configuration of the aquifer and of the karst conduit system, and also on the distribution of hydraulic parameters.

If the heterogeneity of a karst system is lower than a quantitatively defined threshold, the conduits cannot effectively drain the low-permeability matrix. Such systems behave similarly to a porous system, as the drainage process is dependent on the entire aquifer area, and equivalent hydraulic parameters. However, the geometric factor (γ) is different from that of an equivalent porous system, and thus different formula is required to express the recession coefficient. This flow condition has been defined as *conduit-influenced flow regime (CIFR)*, and is typical during the baseflow of fissured systems or weakly karstified systems, defined as *early karst systems*. The recession coefficient of conduit-influenced systems can be expressed as follows

$$\alpha \approx \frac{2}{3} \frac{K_c f}{S_m A}$$

where a regular perpendicular conduit distribution is assumed.

If the heterogeneity of a karst system exceeds the threshold value, karst conduits do not longer influence the drainage of individual low-permeability blocks. Conduits behave as fix-head boundary conditions, and the drainage process is controlled by the low-permeability blocks alone. This flow condition has been defined as a *matrix-restrained flow regime (MRFR)*, and is typical for highly karstified systems during baseflow periods. These systems have been referred to as *mature karst systems*. The baseflow recession coefficient of 2D matrix-restrained systems can be expressed as follows:

$$\alpha = \frac{2\pi^2 T_m f^2}{S_m}$$

The validity of equivalent models is strongly restricted. While equivalent discrete-continuum models may lead to adequate recession coefficients in the conduit-influenced baseflow domain (fissured systems and early karst systems), there is only one parameter configuration that yields appropriate recession coefficient in the matrix-restrained baseflow domain (mature karst systems). Consequently, the applicability of equivalent discrete-continuum models is precluded in

the case of mature karst systems. The global response of equivalent porous medium models corresponds to the transition between matrix-restrained and conduit-influenced flow, and thus the direct application of equivalent porous medium models corrupts simulated global response both in the case of mature karsts and early (or fissured) systems.

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